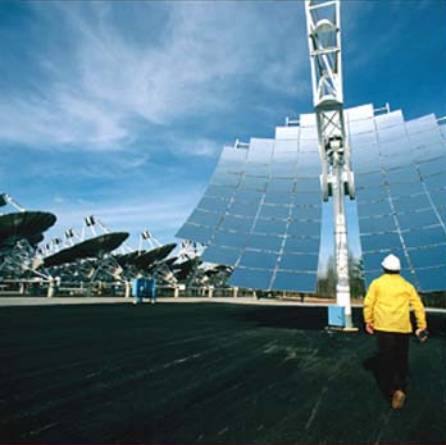


Fifth edition



HIGHER ENGINEERING MATHEMATICS

John Bird



Newnes

HIGHER ENGINEERING MATHEMATICS

In memory of Elizabeth

Higher Engineering Mathematics

Fifth Edition

John Bird, BSc(Hons), CMath, FIMA, FIET, CEng, MIEE, CSci, FCollP, FIIE



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Newnes

Newnes
An imprint of Elsevier
Linacre House, Jordan Hill, Oxford OX2 8DP
30 Corporate Drive, Suite 400, Burlington, MA01803, USA

First published 1993
Second edition 1995
Third edition 1999
Reprinted 2000 (twice), 2001, 2002, 2003
Fourth edition 2004
Fifth edition 2006

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British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

Library of Congress Cataloging-in-Publication Data

A catalog record for this book is available from the Library of Congress

ISBN 13: 9-78-0-75-068152-0

ISBN 10: 0-75-068152-7

For information on all Newnes publications
visit our website at books.elsevier.com

Typeset by Charon Tec Ltd, Chennai, India

www.charontec.com

Printed and bound in Great Britain

06 07 08 09 10 10 9 8 7 6 5 4 3 2 1

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Contents

Preface xv

Syllabus guidance xvii

Section A: Number and Algebra 1

1 Algebra 1

- 1.1 Introduction 1
- 1.2 Revision of basic laws 1
- 1.3 Revision of equations 3
- 1.4 Polynomial division 6
- 1.5 The factor theorem 8
- 1.6 The remainder theorem 10

2 Inequalities 12

- 2.1 Introduction to inequalities 12
- 2.2 Simple inequalities 12
- 2.3 Inequalities involving a modulus 13
- 2.4 Inequalities involving quotients 14
- 2.5 Inequalities involving square functions 15
- 2.6 Quadratic inequalities 16

3 Partial fractions 18

- 3.1 Introduction to partial fractions 18
- 3.2 Worked problems on partial fractions with linear factors 18
- 3.3 Worked problems on partial fractions with repeated linear factors 21
- 3.4 Worked problems on partial fractions with quadratic factors 22

4 Logarithms and exponential functions 24

- 4.1 Introduction to logarithms 24
- 4.2 Laws of logarithms 24
- 4.3 Indicial equations 26
- 4.4 Graphs of logarithmic functions 27
- 4.5 The exponential function 28
- 4.6 The power series for e^x 29
- 4.7 Graphs of exponential functions 31
- 4.8 Napierian logarithms 33
- 4.9 Laws of growth and decay 35
- 4.10 Reduction of exponential laws to linear form 38

5 Hyperbolic functions 41

- 5.1 Introduction to hyperbolic functions 41
- 5.2 Graphs of hyperbolic functions 43
- 5.3 Hyperbolic identities 44
- 5.4 Solving equations involving hyperbolic functions 47
- 5.5 Series expansions for $\cosh x$ and $\sinh x$ 48

Assignment 1 50

6 Arithmetic and geometric progressions 51

- 6.1 Arithmetic progressions 51
- 6.2 Worked problems on arithmetic progressions 51
- 6.3 Further worked problems on arithmetic progressions 52
- 6.4 Geometric progressions 54
- 6.5 Worked problems on geometric progressions 55
- 6.6 Further worked problems on geometric progressions 56

7 The binomial series 58

- 7.1 Pascal's triangle 58
- 7.2 The binomial series 59
- 7.3 Worked problems on the binomial series 59
- 7.4 Further worked problems on the binomial series 61
- 7.5 Practical problems involving the binomial theorem 64

8 Maclaurin's series 67

- 8.1 Introduction 67
- 8.2 Derivation of Maclaurin's theorem 67
- 8.3 Conditions of Maclaurin's series 67
- 8.4 Worked problems on Maclaurin's series 68
- 8.5 Numerical integration using Maclaurin's series 71
- 8.6 Limiting values 72

Assignment 2 75

9 Solving equations by iterative methods	76	13 Cartesian and polar co-ordinates	133
9.1 Introduction to iterative methods	76	13.1 Introduction	133
9.2 The bisection method	76	13.2 Changing from Cartesian into polar co-ordinates	133
9.3 An algebraic method of successive approximations	80	13.3 Changing from polar into Cartesian co-ordinates	135
9.4 The Newton-Raphson method	83	13.4 Use of $R \rightarrow P$ and $P \rightarrow R$ functions on calculators	136
10 Computer numbering systems	86	14 The circle and its properties	137
10.1 Binary numbers	86	14.1 Introduction	137
10.2 Conversion of binary to denary	86	14.2 Properties of circles	137
10.3 Conversion of denary to binary	87	14.3 Arc length and area of a sector	138
10.4 Conversion of denary to binary via octal	88	14.4 Worked problems on arc length and sector of a circle	139
10.5 Hexadecimal numbers	90	14.5 The equation of a circle	140
11 Boolean algebra and logic circuits	94	14.6 Linear and angular velocity	142
11.1 Boolean algebra and switching circuits	94	14.7 Centripetal force	144
11.2 Simplifying Boolean expressions	99	Assignment 4	146
11.3 Laws and rules of Boolean algebra	99	15 Trigonometric waveforms	148
11.4 De Morgan's laws	101	15.1 Graphs of trigonometric functions	148
11.5 Karnaugh maps	102	15.2 Angles of any magnitude	148
11.6 Logic circuits	106	15.3 The production of a sine and cosine wave	151
11.7 Universal logic gates	110	15.4 Sine and cosine curves	152
Assignment 3	114	15.5 Sinusoidal form $A \sin(\omega t \pm \alpha)$	157
Section B: Geometry and trigonometry	115	15.6 Harmonic synthesis with complex waveforms	160
12 Introduction to trigonometry	115	16 Trigonometric identities and equations	166
12.1 Trigonometry	115	16.1 Trigonometric identities	166
12.2 The theorem of Pythagoras	115	16.2 Worked problems on trigonometric identities	166
12.3 Trigonometric ratios of acute angles	116	16.3 Trigonometric equations	167
12.4 Solution of right-angled triangles	118	16.4 Worked problems (i) on trigonometric equations	168
12.5 Angles of elevation and depression	119	16.5 Worked problems (ii) on trigonometric equations	169
12.6 Evaluating trigonometric ratios	121	16.6 Worked problems (iii) on trigonometric equations	170
12.7 Sine and cosine rules	124	16.7 Worked problems (iv) on trigonometric equations	171
12.8 Area of any triangle	125	17 The relationship between trigonometric and hyperbolic functions	173
12.9 Worked problems on the solution of triangles and finding their areas	125	17.1 The relationship between trigonometric and hyperbolic functions	173
12.10 Further worked problems on solving triangles and finding their areas	126	17.2 Hyperbolic identities	174
12.11 Practical situations involving trigonometry	128		
12.12 Further practical situations involving trigonometry	130		

18 Compound angles 176

- 18.1 Compound angle formulae 176
- 18.2 Conversion of $a \sin \omega t + b \cos \omega t$ into $R \sin(\omega t + \alpha)$ 178
- 18.3 Double angles 182
- 18.4 Changing products of sines and cosines into sums or differences 183
- 18.5 Changing sums or differences of sines and cosines into products 184
- 18.6 Power waveforms in a.c. circuits 185

Assignment 5 189**Section C: Graphs 191****19 Functions and their curves 191**

- 19.1 Standard curves 191
- 19.2 Simple transformations 194
- 19.3 Periodic functions 199
- 19.4 Continuous and discontinuous functions 199
- 19.5 Even and odd functions 199
- 19.6 Inverse functions 201
- 19.7 Asymptotes 203
- 19.8 Brief guide to curve sketching 209
- 19.9 Worked problems on curve sketching 210

20 Irregular areas, volumes and mean values of waveforms 216

- 20.1 Areas of irregular figures 216
- 20.2 Volumes of irregular solids 218
- 20.3 The mean or average value of a waveform 219

Section D: Vector geometry 225**21 Vectors, phasors and the combination of waveforms 225**

- 21.1 Introduction 225
- 21.2 Vector addition 225
- 21.3 Resolution of vectors 227
- 21.4 Vector subtraction 229
- 21.5 Relative velocity 231
- 21.6 Combination of two periodic functions 232

22 Scalar and vector products 237

- 22.1 The unit triad 237
- 22.2 The scalar product of two vectors 238

- 22.3 Vector products 241
- 22.4 Vector equation of a line 245

Assignment 6 247**Section E: Complex numbers 249****23 Complex numbers 249**

- 23.1 Cartesian complex numbers 249
- 23.2 The Argand diagram 250
- 23.3 Addition and subtraction of complex numbers 250
- 23.4 Multiplication and division of complex numbers 251
- 23.5 Complex equations 253
- 23.6 The polar form of a complex number 254
- 23.7 Multiplication and division in polar form 256
- 23.8 Applications of complex numbers 257

24 De Moivre's theorem 261

- 24.1 Introduction 261
- 24.2 Powers of complex numbers 261
- 24.3 Roots of complex numbers 262
- 24.4 The exponential form of a complex number 264

Section F: Matrices and Determinants 267**25 The theory of matrices and determinants 267**

- 25.1 Matrix notation 267
- 25.2 Addition, subtraction and multiplication of matrices 267
- 25.3 The unit matrix 271
- 25.4 The determinant of a 2 by 2 matrix 271
- 25.5 The inverse or reciprocal of a 2 by 2 matrix 272
- 25.6 The determinant of a 3 by 3 matrix 273
- 25.7 The inverse or reciprocal of a 3 by 3 matrix 274

26 The solution of simultaneous equations by matrices and determinants 277

- 26.1 Solution of simultaneous equations by matrices 277
- 26.2 Solution of simultaneous equations by determinants 279

- 26.3 Solution of simultaneous equations using Cramers rule 283
- 26.4 Solution of simultaneous equations using the Gaussian elimination method 284

Assignment 7 286

Section G: Differential calculus 287

27 Methods of differentiation 287

- 27.1 The gradient of a curve 287
- 27.2 Differentiation from first principles 288
- 27.3 Differentiation of common functions 288
- 27.4 Differentiation of a product 292
- 27.5 Differentiation of a quotient 293
- 27.6 Function of a function 295
- 27.7 Successive differentiation 296

28 Some applications of differentiation 298

- 28.1 Rates of change 298
- 28.2 Velocity and acceleration 299
- 28.3 Turning points 302
- 28.4 Practical problems involving maximum and minimum values 306
- 28.5 Tangents and normals 310
- 28.6 Small changes 311

29 Differentiation of parametric equations 314

- 29.1 Introduction to parametric equations 314
- 29.2 Some common parametric equations 314
- 29.3 Differentiation in parameters 314
- 29.4 Further worked problems on differentiation of parametric equations 316

30 Differentiation of implicit functions 319

- 30.1 Implicit functions 319
- 30.2 Differentiating implicit functions 319
- 30.3 Differentiating implicit functions containing products and quotients 320
- 30.4 Further implicit differentiation 321

31 Logarithmic differentiation 324

- 31.1 Introduction to logarithmic differentiation 324
- 31.2 Laws of logarithms 324

- 31.3 Differentiation of logarithmic functions 324
- 31.4 Differentiation of $[f(x)]^x$ 327

Assignment 8 329

32 Differentiation of hyperbolic functions 330

- 32.1 Standard differential coefficients of hyperbolic functions 330
- 32.2 Further worked problems on differentiation of hyperbolic functions 331

33 Differentiation of inverse trigonometric and hyperbolic functions 332

- 33.1 Inverse functions 332
- 33.2 Differentiation of inverse trigonometric functions 332
- 33.3 Logarithmic forms of the inverse hyperbolic functions 337
- 33.4 Differentiation of inverse hyperbolic functions 338

34 Partial differentiation 343

- 34.1 Introduction to partial derivatives 343
- 34.2 First order partial derivatives 343
- 34.3 Second order partial derivatives 346

35 Total differential, rates of change and small changes 349

- 35.1 Total differential 349
- 35.2 Rates of change 350
- 35.3 Small changes 352

36 Maxima, minima and saddle points for functions of two variables 355

- 36.1 Functions of two independent variables 355
- 36.2 Maxima, minima and saddle points 355
- 36.3 Procedure to determine maxima, minima and saddle points for functions of two variables 356
- 36.4 Worked problems on maxima, minima and saddle points for functions of two variables 357
- 36.5 Further worked problems on maxima, minima and saddle points for functions of two variables 359

Assignment 9 365

Section H: Integral calculus 367

37 Standard integration 367

- 37.1 The process of integration 367
- 37.2 The general solution of integrals of the form ax^n 367
- 37.3 Standard integrals 367
- 37.4 Definite integrals 371

38 Some applications of integration 374

- 38.1 Introduction 374
- 38.2 Areas under and between curves 374
- 38.3 Mean and r.m.s. values 376
- 38.4 Volumes of solids of revolution 377
- 38.5 Centroids 378
- 38.6 Theorem of Pappus 380
- 38.7 Second moments of area of regular sections 382

39 Integration using algebraic substitutions 391

- 39.1 Introduction 391
- 39.2 Algebraic substitutions 391
- 39.3 Worked problems on integration using algebraic substitutions 391
- 39.4 Further worked problems on integration using algebraic substitutions 393
- 39.5 Change of limits 393

Assignment 10 396

40 Integration using trigonometric and hyperbolic substitutions 397

- 40.1 Introduction 397
- 40.2 Worked problems on integration of $\sin^2 x$, $\cos^2 x$, $\tan^2 x$ and $\cot^2 x$ 397
- 40.3 Worked problems on powers of sines and cosines 399
- 40.4 Worked problems on integration of products of sines and cosines 400
- 40.5 Worked problems on integration using the $\sin \theta$ substitution 401
- 40.6 Worked problems on integration using $\tan \theta$ substitution 403
- 40.7 Worked problems on integration using the $\sinh \theta$ substitution 403
- 40.8 Worked problems on integration using the $\cosh \theta$ substitution 405

41 Integration using partial fractions 408

- 41.1 Introduction 408
- 41.2 Worked problems on integration using partial fractions with linear factors 408
- 41.3 Worked problems on integration using partial fractions with repeated linear factors 409
- 41.4 Worked problems on integration using partial fractions with quadratic factors 410

42 The $t = \tan \frac{\theta}{2}$ substitution 413

- 42.1 Introduction 413
- 42.2 Worked problems on the $t = \tan \frac{\theta}{2}$ substitution 413
- 42.3 Further worked problems on the $t = \tan \frac{\theta}{2}$ substitution 415

Assignment 11 417

43 Integration by parts 418

- 43.1 Introduction 418
- 43.2 Worked problems on integration by parts 418
- 43.3 Further worked problems on integration by parts 420

44 Reduction formulae 424

- 44.1 Introduction 424
- 44.2 Using reduction formulae for integrals of the form $\int x^n e^x dx$ 424
- 44.3 Using reduction formulae for integrals of the form $\int x^n \cos x dx$ and $\int x^n \sin x dx$ 425
- 44.4 Using reduction formulae for integrals of the form $\int \sin^n x dx$ and $\int \cos^n x dx$ 427
- 44.5 Further reduction formulae 430

45 Numerical integration 433

- 45.1 Introduction 433
- 45.2 The trapezoidal rule 433
- 45.3 The mid-ordinate rule 435
- 45.4 Simpson's rule 437

Assignment 12 441

Section I: Differential equations 443

46 Solution of first order differential equations by separation of variables 443

- 46.1 Family of curves 443
- 46.2 Differential equations 444
- 46.3 The solution of equations of the form $\frac{dy}{dx} = f(x)$ 444
- 46.4 The solution of equations of the form $\frac{dy}{dx} = f(y)$ 446
- 46.5 The solution of equations of the form $\frac{dy}{dx} = f(x) \cdot f(y)$ 448

47 Homogeneous first order differential equations 451

- 47.1 Introduction 451
- 47.2 Procedure to solve differential equations of the form $P \frac{dy}{dx} = Q$ 451
- 47.3 Worked problems on homogeneous first order differential equations 451
- 47.4 Further worked problems on homogeneous first order differential equations 452

48 Linear first order differential equations 455

- 48.1 Introduction 455
- 48.2 Procedure to solve differential equations of the form $\frac{dy}{dx} + Py = Q$ 455
- 48.3 Worked problems on linear first order differential equations 456
- 48.4 Further worked problems on linear first order differential equations 457

49 Numerical methods for first order differential equations 460

- 49.1 Introduction 460
- 49.2 Euler's method 460
- 49.3 Worked problems on Euler's method 461
- 49.4 An improved Euler method 465
- 49.5 The Runge-Kutta method 469

Assignment 13 474

50 Second order differential equations of the

form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$ 475

- 50.1 Introduction 475
- 50.2 Procedure to solve differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$ 475
- 50.3 Worked problems on differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$ 476
- 50.4 Further worked problems on practical differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$ 478

51 Second order differential equations of the

form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$ 481

- 51.1 Complementary function and particular integral 481
- 51.2 Procedure to solve differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$ 481
- 51.3 Worked problems on differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$ where $f(x)$ is a constant or polynomial 482
- 51.4 Worked problems on differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$ where $f(x)$ is an exponential function 484
- 51.5 Worked problems on differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$ where $f(x)$ is a sine or cosine function 486
- 51.6 Worked problems on differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$ where $f(x)$ is a sum or a product 488

52 Power series methods of solving ordinary differential equations 491

- 52.1 Introduction 491
- 52.2 Higher order differential coefficients as series 491
- 52.3 Leibniz's theorem 493
- 52.4 Power series solution by the Leibniz–Maclaurin method 495
- 52.5 Power series solution by the Frobenius method 498
- 52.6 Bessel's equation and Bessel's functions 504
- 52.7 Legendre's equation and Legendre polynomials 509

53 An introduction to partial differential equations 512

- 53.1 Introduction 512
- 53.2 Partial integration 512
- 53.3 Solution of partial differential equations by direct partial integration 513
- 53.4 Some important engineering partial differential equations 515
- 53.5 Separating the variables 515
- 53.6 The wave equation 516
- 53.7 The heat conduction equation 520
- 53.8 Laplace's equation 522

Assignment 14 525**Section J: Statistics and probability 527****54 Presentation of statistical data 527**

- 54.1 Some statistical terminology 527
- 54.2 Presentation of ungrouped data 528
- 54.3 Presentation of grouped data 532

55 Measures of central tendency and dispersion 538

- 55.1 Measures of central tendency 538
- 55.2 Mean, median and mode for discrete data 538
- 55.3 Mean, median and mode for grouped data 539
- 55.4 Standard deviation 541
- 55.5 Quartiles, deciles and percentiles 543

56 Probability 545

- 56.1 Introduction to probability 545
- 56.2 Laws of probability 545
- 56.3 Worked problems on probability 546
- 56.4 Further worked problems on probability 548

Assignment 15 551**57 The binomial and Poisson distributions 553**

- 57.1 The binomial distribution 553
- 57.2 The Poisson distribution 556

58 The normal distribution 559

- 58.1 Introduction to the normal distribution 559
- 58.2 Testing for a normal distribution 563

59 Linear correlation 567

- 59.1 Introduction to linear correlation 567
- 59.2 The product-moment formula for determining the linear correlation coefficient 567
- 59.3 The significance of a coefficient of correlation 568
- 59.4 Worked problems on linear correlation 568

60 Linear regression 571

- 60.1 Introduction to linear regression 571
- 60.2 The least-squares regression lines 571
- 60.3 Worked problems on linear regression 572

Assignment 16 576**61 Sampling and estimation theories 577**

- 61.1 Introduction 577
- 61.2 Sampling distributions 577
- 61.3 The sampling distribution of the means 577
- 61.4 The estimation of population parameters based on a large sample size 581
- 61.5 Estimating the mean of a population based on a small sample size 586

62 Significance testing 590

- 62.1 Hypotheses 590
- 62.2 Type I and Type II errors 590

- 62.3 Significance tests for population means 597
- 62.4 Comparing two sample means 602
- 63 Chi-square and distribution-free tests 607**
- 63.1 Chi-square values 607
- 63.2 Fitting data to theoretical distributions 608
- 63.3 Introduction to distribution-free tests 613
- 63.4 The sign test 614
- 63.5 Wilcoxon signed-rank test 616
- 63.6 The Mann-Whitney test 620
- Assignment 17 625**
- Section K: Laplace transforms 627**
- 64 Introduction to Laplace transforms 627**
- 64.1 Introduction 627
- 64.2 Definition of a Laplace transform 627
- 64.3 Linearity property of the Laplace transform 627
- 64.4 Laplace transforms of elementary functions 627
- 64.5 Worked problems on standard Laplace transforms 629
- 65 Properties of Laplace transforms 632**
- 65.1 The Laplace transform of $e^{at}f(t)$ 632
- 65.2 Laplace transforms of the form $e^{at}f(t)$ 632
- 65.3 The Laplace transforms of derivatives 634
- 65.4 The initial and final value theorems 636
- 66 Inverse Laplace transforms 638**
- 66.1 Definition of the inverse Laplace transform 638
- 66.2 Inverse Laplace transforms of simple functions 638
- 66.3 Inverse Laplace transforms using partial fractions 640
- 66.4 Poles and zeros 642
- 67 The solution of differential equations using Laplace transforms 645**
- 67.1 Introduction 645
- 67.2 Procedure to solve differential equations by using Laplace transforms 645
- 67.3 Worked problems on solving differential equations using Laplace transforms 645
- 68 The solution of simultaneous differential equations using Laplace transforms 650**
- 68.1 Introduction 650
- 68.2 Procedure to solve simultaneous differential equations using Laplace transforms 650
- 68.3 Worked problems on solving simultaneous differential equations by using Laplace transforms 650
- Assignment 18 655**
- Section L: Fourier series 657**
- 69 Fourier series for periodic functions of period 2π 657**
- 69.1 Introduction 657
- 69.2 Periodic functions 657
- 69.3 Fourier series 657
- 69.4 Worked problems on Fourier series of periodic functions of period 2π 658
- 70 Fourier series for a non-periodic function over range 2π 663**
- 70.1 Expansion of non-periodic functions 663
- 70.2 Worked problems on Fourier series of non-periodic functions over a range of 2π 663
- 71 Even and odd functions and half-range Fourier series 669**
- 71.1 Even and odd functions 669
- 71.2 Fourier cosine and Fourier sine series 669
- 71.3 Half-range Fourier series 672
- 72 Fourier series over any range 676**
- 72.1 Expansion of a periodic function of period L 676
- 72.2 Half-range Fourier series for functions defined over range L 680

73 A numerical method of harmonic analysis 683	74.3 Complex coefficients 691
73.1 Introduction 683	74.4 Symmetry relationships 695
73.2 Harmonic analysis on data given in tabular or graphical form 683	74.5 The frequency spectrum 698
73.3 Complex waveform considerations 686	74.6 Phasors 699
74 The complex or exponential form of a Fourier series 690	Assignment 19 704
74.1 Introduction 690	Essential formulae 705
74.2 Exponential or complex notation 690	Index 721

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Preface

This **fifth edition** of '*Higher Engineering Mathematics*' covers essential mathematical material suitable for students studying **Degrees, Foundation Degrees, Higher National Certificate and Diploma courses in Engineering disciplines**.

In this edition the material has been re-ordered into the following **twelve convenient categories**: number and algebra, geometry and trigonometry, graphs, vector geometry, complex numbers, matrices and determinants, differential calculus, integral calculus, differential equations, statistics and probability, Laplace transforms and Fourier series. **New material** has been added on inequalities, differentiation of parametric equations, the $t = \tan \theta/2$ substitution and homogeneous first order differential equations. Another new feature is that a **free Internet download** is available to lecturers of a sample of solutions (over 1000) of the further problems contained in the book.

The primary aim of the material in this text is to provide the fundamental analytical and underpinning knowledge and techniques needed to successfully complete scientific and engineering principles modules of Degree, Foundation Degree and Higher National Engineering programmes. The material has been designed to enable students to use techniques learned for the analysis, modelling and solution of realistic engineering problems at Degree and Higher National level. It also aims to provide some of the more advanced knowledge required for those wishing to pursue careers in mechanical engineering, aeronautical engineering, electronics, communications engineering, systems engineering and all variants of control engineering.

In *Higher Engineering Mathematics 5th Edition*, theory is introduced in each chapter by a full outline of essential definitions, formulae, laws, procedures etc. The theory is kept to a minimum, for **problem solving** is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.

Access to software packages such as Maple, Mathematica and Derive, or a graphics calculator, will enhance understanding of some of the topics in this text.

Each topic considered in the text is presented in a way that assumes in the reader only the knowledge attained in BTEC National Certificate/Diploma in an Engineering discipline and Advanced GNVQ in Engineering/Manufacture.

'*Higher Engineering Mathematics*' provides a follow-up to '*Engineering Mathematics*'.

This textbook contains some **1000 worked problems**, followed by over **1750 further problems (with answers)**, arranged within **250 Exercises**. Some **460 line diagrams** further enhance understanding.

A **sample of worked solutions** to over 1000 of the further problems has been prepared and can be **accessed by lecturers free via the Internet** (see below).

At the end of the text, a list of **Essential Formulae** is included for convenience of reference.

At intervals throughout the text are some **19 Assignments** to check understanding. For example, Assignment 1 covers the material in chapters 1 to 5, Assignment 2 covers the material in chapters 6 to 8, Assignment 3 covers the material in chapters 9 to 11, and so on. An **Instructor's Manual**, containing full solutions to the Assignments, is available free to lecturers adopting this text (see below).

'**Learning by example**' is at the heart of '*Higher Engineering Mathematics 5th Edition*'.

JOHN BIRD

Royal Naval School of Marine Engineering, HMS Sultan,
formerly University of Portsmouth
and Highbury College, Portsmouth

Free web downloads

Extra material available on the Internet

It is recognised that the **level of understanding of algebra** on entry to higher courses is often inadequate. Since algebra provides the basis of so much of higher engineering studies, it is a situation that often needs urgent attention. Lack of space has prevented the inclusion of more basic algebra topics in this textbook;

it is for this reason that some algebra topics – solution of simple, simultaneous and quadratic equations and transposition of formulae have been made available to all via the Internet. Also included is a Remedial Algebra Assignment to test understanding.

To access the Algebra material visit: <http://books.elsevier.com/companions/0750681527>

Sample of Worked Solutions to Exercises

Within the text are some 1750 further problems arranged within 250 Exercises. A sample of over 1000 worked solutions has been prepared and is available for lecturers only at <http://www.textbooks.elsevier.com>

Instructor's manual

This provides full worked solutions and mark scheme for all 19 Assignments in this book,

together with solutions to the Remedial Algebra Assignment mentioned above. The material is available to lecturers only and is available at <http://www.textbooks.elsevier.com>

To access the lecturer material on the textbook website please go to <http://www.textbooks.elsevier.com> and search for the book and click on the 'manual' link. If you do not have an account on [textbooks.elsevier.com](http://www.textbooks.elsevier.com) already, you will need to register and request access to the book's subject area. If you already have an account on [textbooks](http://www.textbooks.elsevier.com), but do not have access to the right subject area, please follow the 'request access' link at the top of the subject area homepage.

Syllabus guidance

This textbook is written for **undergraduate engineering degree and foundation degree courses**; however, it is also most appropriate for **HNC/D studies** and three syllabuses are covered. The appropriate chapters for these three syllabuses are shown in the table below.

Chapter		Analytical Methods for Engineers	Further Analytical Methods for Engineers	Engineering Mathematics
1.	Algebra	×		
2.	Inequalities			
3.	Partial fractions	×		
4.	Logarithms and exponential functions	×		
5.	Hyperbolic functions	×		
6.	Arithmetic and geometric progressions	×		
7.	The binomial series	×		
8.	Maclaurin's series	×		
9.	Solving equations by iterative methods		×	
10.	Computer numbering systems		×	
11.	Boolean algebra and logic circuits		×	
12.	Introduction to trigonometry	×		
13.	Cartesian and polar co-ordinates	×		
14.	The circle and its properties	×		
15.	Trigonometric waveforms	×		
16.	Trigonometric identities and equations	×		
17.	The relationship between trigonometric and hyperbolic functions	×		
18.	Compound angles	×		
19.	Functions and their curves		×	
20.	Irregular areas, volumes and mean value of waveforms		×	
21.	Vectors, phasors and the combination of waveforms		×	
22.	Scalar and vector products		×	
23.	Complex numbers		×	
24.	De Moivre's theorem		×	
25.	The theory of matrices and determinants		×	
26.	The solution of simultaneous equations by matrices and determinants		×	
27.	Methods of differentiation	×		
28.	Some applications of differentiation	×		
29.	Differentiation of parametric equations			
30.	Differentiation of implicit functions	×		
31.	Logarithmic differentiation	×		
32.	Differentiation of hyperbolic functions	×		
33.	Differentiation of inverse trigonometric and hyperbolic functions	×		
34.	Partial differentiation			×

(Continued)

xviii SYLLABUS GUIDANCE

Chapter	Analytical Methods for Engineers	Further Analytical Methods for Engineers	Engineering Mathematics
35.	Total differential, rates of change and small changes		×
36.	Maxima, minima and saddle points for functions of two variables		×
37.	Standard integration	×	
38.	Some applications of integration	×	
39.	Integration using algebraic substitutions	×	
40.	Integration using trigonometric and hyperbolic substitutions	×	
41.	Integration using partial fractions	×	
42.	The $t = \tan \theta/2$ substitution		
43.	Integration by parts	×	
44.	Reduction formulae	×	
45.	Numerical integration		×
46.	Solution of first order differential equations by separation of variables		×
47.	Homogeneous first order differential equations		
48.	Linear first order differential equations		×
49.	Numerical methods for first order differential equations		×
50.	Second order differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = 0$		×
51.	Second order differential equations of the form $a \frac{d^2y}{dx^2} + b \frac{dy}{dx} + cy = f(x)$		×
52.	Power series methods of solving ordinary differential equations		×
53.	An introduction to partial differential equations		×
54.	Presentation of statistical data	×	
55.	Measures of central tendency and dispersion	×	
56.	Probability	×	
57.	The binomial and Poisson distributions	×	
58.	The normal distribution	×	
59.	Linear correlation	×	
60.	Linear regression	×	
61.	Sampling and estimation theories	×	
62.	Significance testing	×	
63.	Chi-square and distribution-free tests	×	
64.	Introduction to Laplace transforms		×
65.	Properties of Laplace transforms		×
66.	Inverse Laplace transforms		×
67.	Solution of differential equations using Laplace transforms		×
68.	The solution of simultaneous differential equations using Laplace transforms		×
69.	Fourier series for periodic functions of period 2π		×
70.	Fourier series for non-periodic functions over range 2π		×
71.	Even and odd functions and half-range Fourier series		×
72.	Fourier series over any range		×
73.	A numerical method of harmonic analysis		×
74.	The complex or exponential form of a Fourier series		×

1

Algebra

1.1 Introduction

In this chapter, polynomial division and the factor and remainder theorems are explained (in Sections 1.4 to 1.6). However, before this, some essential algebra revision on basic laws and equations is included.

For further Algebra revision, go to website:
<http://books.elsevier.com/companions/0750681527>

1.2 Revision of basic laws

(a) Basic operations and laws of indices

The laws of indices are:

- (i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$
 (iii) $(a^m)^n = a^{m \times n}$ (iv) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$
 (v) $a^{-n} = \frac{1}{a^n}$ (vi) $a^0 = 1$

Problem 1. Evaluate $4a^2bc^3 - 2ac$ when $a = 2$, $b = \frac{1}{2}$ and $c = 1\frac{1}{2}$

$$\begin{aligned} 4a^2bc^3 - 2ac &= 4(2)^2 \left(\frac{1}{2}\right) \left(\frac{3}{2}\right)^3 - 2(2) \left(\frac{3}{2}\right) \\ &= \frac{4 \times 2 \times 2 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 2} - \frac{12}{2} \\ &= 27 - 6 = \mathbf{21} \end{aligned}$$

Problem 2. Multiply $3x + 2y$ by $x - y$.

$$\begin{array}{r} 3x + 2y \\ x - y \\ \hline \text{Multiply by } x \rightarrow 3x^2 + 2xy \\ \text{Multiply by } -y \rightarrow \quad -3xy - 2y^2 \\ \hline \text{Adding gives: } \quad \underline{3x^2 - xy - 2y^2} \end{array}$$

Alternatively,

$$\begin{aligned} (3x + 2y)(x - y) &= 3x^2 - 3xy + 2xy - 2y^2 \\ &= \mathbf{3x^2 - xy - 2y^2} \end{aligned}$$

Problem 3. Simplify $\frac{a^3b^2c^4}{abc^{-2}}$ and evaluate when $a = 3$, $b = \frac{1}{8}$ and $c = 2$.

$$\frac{a^3b^2c^4}{abc^{-2}} = a^{3-1}b^{2-1}c^{4-(-2)} = a^2bc^6$$

When $a = 3$, $b = \frac{1}{8}$ and $c = 2$,

$$a^2bc^6 = (3)^2 \left(\frac{1}{8}\right) (2)^6 = (9) \left(\frac{1}{8}\right) (64) = \mathbf{72}$$

Problem 4. Simplify $\frac{x^2y^3 + xy^2}{xy}$

$$\begin{aligned} \frac{x^2y^3 + xy^2}{xy} &= \frac{x^2y^3}{xy} + \frac{xy^2}{xy} \\ &= x^{2-1}y^{3-1} + x^{1-1}y^{2-1} \\ &= \mathbf{xy^2 + y} \quad \text{or} \quad \mathbf{y(xy + 1)} \end{aligned}$$

Problem 5. Simplify $\frac{(x^2\sqrt{y})(\sqrt{x}\sqrt[3]{y^2})}{(x^5y^3)^{\frac{1}{2}}}$

$$\begin{aligned} \frac{(x^2\sqrt{y})(\sqrt{x}\sqrt[3]{y^2})}{(x^5y^3)^{\frac{1}{2}}} &= \frac{x^2y^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{2}{3}}}{x^{\frac{5}{2}}y^{\frac{3}{2}}} \\ &= x^{2+\frac{1}{2}-\frac{5}{2}}y^{\frac{1}{2}+\frac{2}{3}-\frac{3}{2}} \\ &= x^0y^{-\frac{1}{3}} \\ &= y^{-\frac{1}{3}} \quad \text{or} \quad \frac{1}{y^{\frac{1}{3}}} \quad \text{or} \quad \frac{1}{\sqrt[3]{y}} \end{aligned}$$

2 NUMBER AND ALGEBRA

Now try the following exercise.

Exercise 1 Revision of basic operations and laws of indices

1. Evaluate $2ab + 3bc - abc$ when $a = 2$,
 $b = -2$ and $c = 4$. $[-16]$

2. Find the value of $5pq^2r^3$ when $p = \frac{2}{5}$,
 $q = -2$ and $r = -1$. $[-8]$

3. From $4x - 3y + 2z$ subtract $x + 2y - 3z$.
 $[3x - 5y + 5z]$

4. Multiply $2a - 5b + c$ by $3a + b$.
 $[6a^2 - 13ab + 3ac - 5b^2 + bc]$

5. Simplify $(x^2y^3z)(x^3yz^2)$ and evaluate when
 $x = \frac{1}{2}$, $y = 2$ and $z = 3$. $[x^5y^4z^3, 13\frac{1}{2}]$

6. Evaluate $(a^{\frac{3}{2}}bc^{-3})(a^{\frac{1}{2}}b^{-\frac{1}{2}}c)$ when $a = 3$,
 $b = 4$ and $c = 2$. $[\pm 4\frac{1}{2}]$

7. Simplify $\frac{a^2b + a^3b}{a^2b^2}$ $[\frac{1+a}{b}]$

8. Simplify $\frac{(a^3b^{\frac{1}{2}}c^{-\frac{1}{2}})(ab)^{\frac{1}{3}}}{(\sqrt{a^3}\sqrt{bc})}$
 $[\frac{a^{\frac{11}{6}}b^{\frac{1}{3}}c^{-\frac{3}{2}}}{\sqrt{c^3}} \text{ or } \frac{\sqrt[6]{a^{11}}\sqrt[3]{b}}{\sqrt{c^3}}]$

(b) Brackets, factorization and precedence

Problem 6. Simplify

$$a^2 - (2a - ab) - a(3b + a).$$

$$\begin{aligned} a^2 - (2a - ab) - a(3b + a) \\ &= a^2 - 2a + ab - 3ab - a^2 \\ &= -2a - 2ab \text{ or } -2a(1 + b) \end{aligned}$$

Problem 7. Remove the brackets and simplify the expression:

$$2a - [3\{2(4a - b) - 5(a + 2b)\} + 4a].$$

Removing the innermost brackets gives:

$$2a - [3\{8a - 2b - 5a - 10b\} + 4a]$$

Collecting together similar terms gives:

$$2a - [3\{3a - 12b\} + 4a]$$

Removing the 'curly' brackets gives:

$$2a - [9a - 36b + 4a]$$

Collecting together similar terms gives:

$$2a - [13a - 36b]$$

Removing the square brackets gives:

$$2a - 13a + 36b = -11a + 36b \text{ or } 36b - 11a$$

Problem 8. Factorize (a) $xy - 3xz$
(b) $4a^2 + 16ab^3$ (c) $3a^2b - 6ab^2 + 15ab$.

(a) $xy - 3xz = x(y - 3z)$

(b) $4a^2 + 16ab^3 = 4a(a + 4b^3)$

(c) $3a^2b - 6ab^2 + 15ab = 3ab(a - 2b + 5)$

Problem 9. Simplify $3c + 2c \times 4c + c \div 5c - 8c$.

The order of precedence is division, multiplication, addition and subtraction (sometimes remembered by BODMAS). Hence

$$\begin{aligned} 3c + 2c \times 4c + c \div 5c - 8c \\ &= 3c + 2c \times 4c + \left(\frac{c}{5c}\right) - 8c \\ &= 3c + 8c^2 + \frac{1}{5} - 8c \\ &= 8c^2 - 5c + \frac{1}{5} \text{ or } c(8c - 5) + \frac{1}{5} \end{aligned}$$

Problem 10. Simplify
 $(2a - 3) \div 4a + 5 \times 6 - 3a$.

$$\begin{aligned} (2a - 3) \div 4a + 5 \times 6 - 3a \\ &= \frac{2a - 3}{4a} + 5 \times 6 - 3a \\ &= \frac{2a - 3}{4a} + 30 - 3a \\ &= \frac{2a}{4a} - \frac{3}{4a} + 30 - 3a \\ &= \frac{1}{2} - \frac{3}{4a} + 30 - 3a = 30\frac{1}{2} - \frac{3}{4a} - 3a \end{aligned}$$

Now try the following exercise.

Exercise 2 Further problems on brackets, factorization and precedence

- Simplify $2(p + 3q - r) - 4(r - q + 2p) + p$.
[$-5p + 10q - 6r$]
- Expand and simplify $(x + y)(x - 2y)$.
[$x^2 - xy - 2y^2$]
- Remove the brackets and simplify:
 $24p - [2\{3(5p - q) - 2(p + 2q)\} + 3q]$.
[$11q - 2p$]
- Factorize $21a^2b^2 - 28ab$ [$7ab(3ab - 4)$].
- Factorize $2xy^2 + 6x^2y + 8x^3y$.
[$2xy(y + 3x + 4x^2)$]
- Simplify $2y + 4 \div 6y + 3 \times 4 - 5y$.
[$\frac{2}{3y} - 3y + 12$]
- Simplify $3 \div y + 2 \div y - 1$. [$\frac{5}{y} - 1$]
- Simplify $a^2 - 3ab \times 2a \div 6b + ab$. [ab]

1.3 Revision of equations

(a) Simple equations

Problem 11. Solve $4 - 3x = 2x - 11$.

Since $4 - 3x = 2x - 11$ then $4 + 11 = 2x + 3x$
i.e. $15 = 5x$ from which, $x = \frac{15}{5} = 3$

Problem 12. Solve

$$4(2a - 3) - 2(a - 4) = 3(a - 3) - 1.$$

Removing the brackets gives:

$$8a - 12 - 2a + 8 = 3a - 9 - 1$$

Rearranging gives:

$$8a - 2a - 3a = -9 - 1 + 12 - 8$$

i.e. $3a = -6$

and $a = \frac{-6}{3} = -2$

Problem 13. Solve $\frac{3}{x-2} = \frac{4}{3x+4}$.

By 'cross-multiplying': $3(3x + 4) = 4(x - 2)$

Removing brackets gives: $9x + 12 = 4x - 8$

Rearranging gives: $9x - 4x = -8 - 12$

i.e. $5x = -20$

and $x = \frac{-20}{5} = -4$

Problem 14. Solve $\left(\frac{\sqrt{t}+3}{\sqrt{t}}\right) = 2$.

$$\sqrt{t} \left(\frac{\sqrt{t}+3}{\sqrt{t}}\right) = 2\sqrt{t}$$

i.e. $\sqrt{t} + 3 = 2\sqrt{t}$

and $3 = 2\sqrt{t} - \sqrt{t}$

i.e. $3 = \sqrt{t}$

and $9 = t$

(b) Transposition of formulae

Problem 15. Transpose the formula

$$v = u + \frac{ft}{m}$$

to make f the subject.

$$u + \frac{ft}{m} = v \text{ from which, } \frac{ft}{m} = v - u$$

and $m \left(\frac{ft}{m}\right) = m(v - u)$

i.e. $ft = m(v - u)$

and $f = \frac{m}{t}(v - u)$

Problem 16. The impedance of an a.c. circuit is given by $Z = \sqrt{R^2 + X^2}$. Make the reactance X the subject.

4 NUMBER AND ALGEBRA

$\sqrt{R^2 + X^2} = Z$ and squaring both sides gives
 $R^2 + X^2 = Z^2$, from which,

$$X^2 = Z^2 - R^2 \text{ and reactance } X = \sqrt{Z^2 - R^2}$$

Problem 17. Given that $\frac{D}{d} = \sqrt{\left(\frac{f+p}{f-p}\right)}$,
 express p in terms of D , d and f .

Rearranging gives: $\sqrt{\left(\frac{f+p}{f-p}\right)} = \frac{D}{d}$

Squaring both sides gives: $\frac{f+p}{f-p} = \frac{D^2}{d^2}$

'Cross-multiplying' gives:

$$d^2(f+p) = D^2(f-p)$$

Removing brackets gives:

$$d^2f + d^2p = D^2f - D^2p$$

Rearranging gives: $d^2p + D^2p = D^2f - d^2f$

Factorizing gives: $p(d^2 + D^2) = f(D^2 - d^2)$

and $p = \frac{f(D^2 - d^2)}{(d^2 + D^2)}$

Now try the following exercise.

Exercise 3 Further problems on simple equations and transposition of formulae

In problems 1 to 4 solve the equations

1. $3x - 2 - 5x = 2x - 4$ [$\frac{1}{2}$]

2. $8 + 4(x - 1) - 5(x - 3) = 2(5 - 2x)$ [-3]

3. $\frac{1}{3a-2} + \frac{1}{5a+3} = 0$ [- $\frac{1}{8}$]

4. $\frac{3\sqrt{t}}{1-\sqrt{t}} = -6$ [4]

5. Transpose $y = \frac{3(F-f)}{L}$ for f .

$$\left[f = \frac{3F - yL}{3} \text{ or } f = F - \frac{yL}{3} \right]$$

6. Make l the subject of $t = 2\pi\sqrt{\frac{l}{g}}$
 $\left[l = \frac{t^2 g}{4\pi^2} \right]$

7. Transpose $m = \frac{\mu L}{L + rCR}$ for L .
 $\left[L = \frac{mrCR}{\mu - m} \right]$

8. Make r the subject of the formula
 $\frac{x}{y} = \frac{1+r^2}{1-r^2}$ [$r = \sqrt{\left(\frac{x-y}{x+y}\right)}$]

(c) Simultaneous equations

Problem 18. Solve the simultaneous equations:

$$7x - 2y = 26 \quad (1)$$

$$6x + 5y = 29 \quad (2)$$

$5 \times$ equation (1) gives:

$$35x - 10y = 130 \quad (3)$$

$2 \times$ equation (2) gives:

$$12x + 10y = 58 \quad (4)$$

equation (3) + equation (4) gives:

$$47x + 0 = 188$$

from which, $x = \frac{188}{47} = 4$

Substituting $x = 4$ in equation (1) gives:

$$28 - 2y = 26$$

from which, $28 - 26 = 2y$ and $y = 1$

Problem 19. Solve

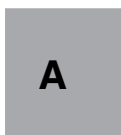
$$\frac{x}{8} + \frac{5}{2} = y \quad (1)$$

$$11 + \frac{y}{3} = 3x \quad (2)$$

$8 \times$ equation (1) gives: $x + 20 = 8y$ (3)

$3 \times$ equation (2) gives: $33 + y = 9x$ (4)

i.e. $x - 8y = -20$ (5)



and $9x - y = 33$ (6)

$8 \times$ equation (6) gives: $72x - 8y = 264$ (7)

Equation (7) – equation (5) gives:

$$71x = 284$$

from which, $x = \frac{284}{71} = 4$

Substituting $x = 4$ in equation (5) gives:

$$4 - 8y = -20$$

from which, $4 + 20 = 8y$ and $y = 3$

(d) Quadratic equations

Problem 20. Solve the following equations by factorization:

(a) $3x^2 - 11x - 4 = 0$
 (b) $4x^2 + 8x + 3 = 0$

(a) The factors of $3x^2$ are $3x$ and x and these are placed in brackets thus:
 $(3x \quad)(x \quad)$

The factors of -4 are $+1$ and -4 or -1 and $+4$, or -2 and $+2$. Remembering that the product of the two inner terms added to the product of the two outer terms must equal $-11x$, the only combination to give this is $+1$ and -4 , i.e.,

$$3x^2 - 11x - 4 = (3x + 1)(x - 4)$$

Thus $(3x + 1)(x - 4) = 0$ hence

either $(3x + 1) = 0$ i.e. $x = -\frac{1}{3}$

or $(x - 4) = 0$ i.e. $x = 4$

(b) $4x^2 + 8x + 3 = (2x + 3)(2x + 1)$

Thus $(2x + 3)(2x + 1) = 0$ hence

either $(2x + 3) = 0$ i.e. $x = -\frac{3}{2}$

or $(2x + 1) = 0$ i.e. $x = -\frac{1}{2}$

Problem 21. The roots of a quadratic equation are $\frac{1}{3}$ and -2 . Determine the equation in x .

If $\frac{1}{3}$ and -2 are the roots of a quadratic equation then,

$$(x - \frac{1}{3})(x + 2) = 0$$

i.e. $x^2 + 2x - \frac{1}{3}x - \frac{2}{3} = 0$

i.e. $x^2 + \frac{5}{3}x - \frac{2}{3} = 0$

or $3x^2 + 5x - 2 = 0$

Problem 22. Solve $4x^2 + 7x + 2 = 0$ giving the answer correct to 2 decimal places.

From the quadratic formula if $ax^2 + bx + c = 0$ then,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hence if $4x^2 + 7x + 2 = 0$

then $x = \frac{-7 \pm \sqrt{7^2 - 4(4)(2)}}{2(4)}$

$$= \frac{-7 \pm \sqrt{17}}{8}$$

$$= \frac{-7 \pm 4.123}{8}$$

$$= \frac{-7 + 4.123}{8} \text{ or } \frac{-7 - 4.123}{8}$$

i.e. $x = -0.36$ or -1.39

Now try the following exercise.

Exercise 4 Further problems on simultaneous and quadratic equations

In problems 1 to 3, solve the simultaneous equations

- $8x - 3y = 51$
 $3x + 4y = 14$ [$x = 6, y = -1$]
- $5a = 1 - 3b$
 $2b + a + 4 = 0$ [$a = 2, b = -3$]
- $\frac{x}{5} + \frac{2y}{3} = \frac{49}{15}$
 $\frac{3x}{7} - \frac{y}{2} + \frac{5}{7} = 0$ [$x = 3, y = 4$]

4. Solve the following quadratic equations by factorization:

- $x^2 + 4x - 32 = 0$
- $8x^2 + 2x - 15 = 0$ [(a) $4, -8$ (b) $\frac{5}{4}, -\frac{3}{2}$]

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