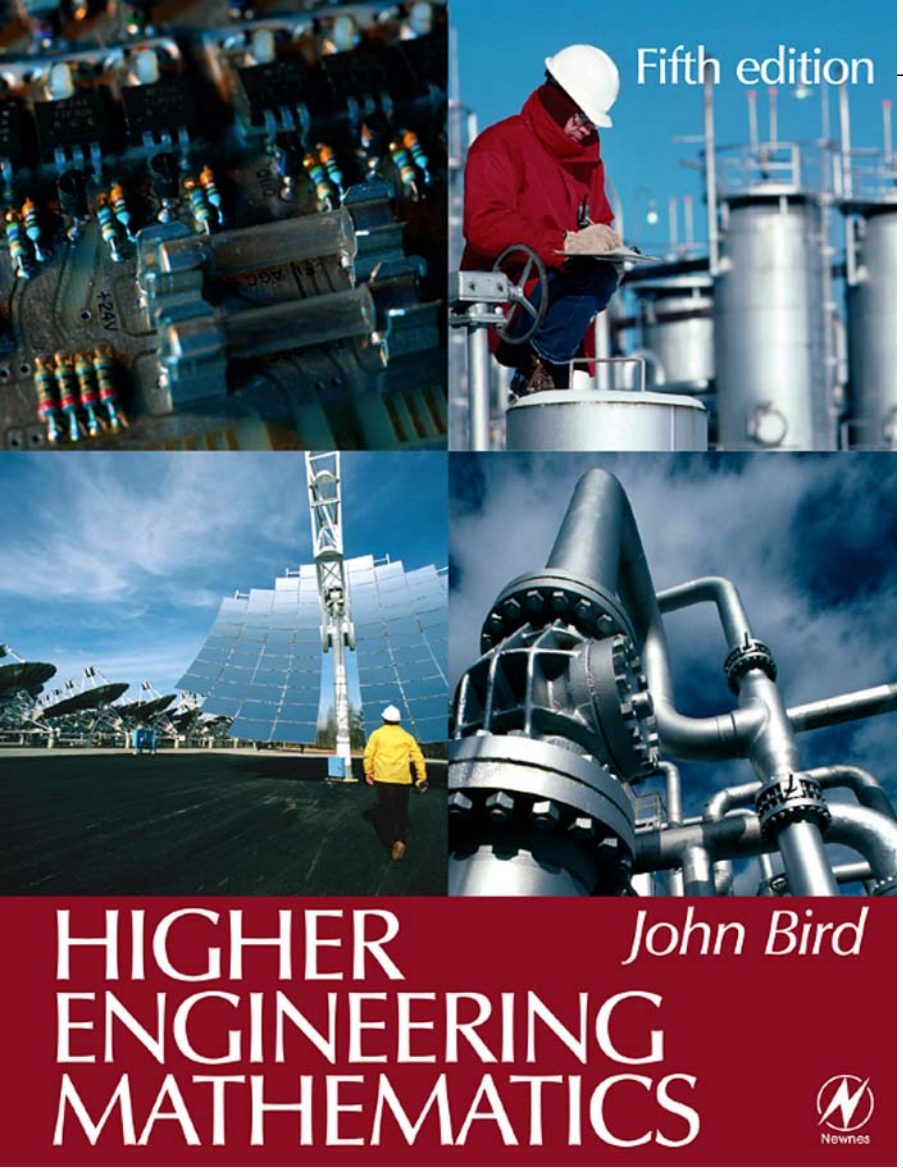

HIGHER ENGINEERING MATHEMATICS

John Bird

Newnes

HIGHER ENGINEERING MATHEMATICS

In memory of Elizabeth

Higher Engineering Mathematics

Fifth Edition

John Bird, BSc(Hons), CMath, FIMA, FIET, CEng, MIEE, CSci, FCollP, FIIE

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Preface

This **fifth edition of** '***Higher Engineering Mathematics***' covers essential mathematical material suitable for students studying **Degrees, Foundation Degrees, Higher National Certificate and Diploma courses in Engineering disciplines.**

In this edition the material has been re-ordered into the following **twelve convenient categories**: number and algebra, geometry and trigonometry, graphs, vector geometry, complex numbers, matrices and determinants, differential calculus, integral calculus, differential equations, statistics and probability, Laplace transforms and Fourier series. **New material** has been added on inequalities, differentiation of parametric equations, the $t = \tan\,\theta/2$ substitution and homogeneous first order differential equations. Another new feature is that a **free Internet download** is available to lecturers of a sample of solutions (over 1000) of the further problems contained in the book.

The primary aim of the material in this text is to provide the fundamental analytical and underpinning knowledge and techniques needed to successfully complete scientific and engineering principles modules of Degree, Foundation Degree and Higher National Engineering programmes. The material has been designed to enable students to use techniques learned for the analysis, modelling and solution of realistic engineering problems at Degree and Higher National level. It also aims to provide some of the more advanced knowledge required for those wishing to pursue careers in mechanical engineering, aeronautical engineering, electronics, communications engineering, systems engineering and all variants of control engineering.

In ***Higher Engineering Mathematics 5th Edition***, theory is introduced in each chapter by a full outline of essential definitions, formulae, laws, procedures etc. The theory is kept to a minimum, for **problem solving** is extensively used to establish and exemplify the theory. It is intended that readers will gain real understanding through seeing problems solved and then through solving similar problems themselves.

Access to software packages such as Maple, Mathematica and Derive, or a graphics calculator, will enhance understanding of some of the topics in this text.

Each topic considered in the text is presented in a way that assumes in the reader only the knowledge attained in BTEC National Certificate/Diploma in an Engineering discipline and Advanced GNVQ in Engineering/Manufacture.

'*Higher Engineering Mathematics*' provides a follow-up to '*Engineering Mathematics*'.

This textbook contains some **1000 worked problems**, followed by over **1750 further problems (with answers)**, arranged within **250 Exercises**. Some **460 line diagrams** further enhance understanding.

A **sample of worked solutions** to over 1000 of the further problems has been prepared and can be **accessed by lecturers free via the Internet** (see below).

At the end of the text, a list of **Essential Formulae** is included for convenience of reference.

At intervals throughout the text are some **19 Assignments** to check understanding. For example, Assignment 1 covers the material in chapters 1 to 5, Assignment 2 covers the material in chapters 6 to 8, Assignment 3 covers the material in chapters 9 to 11, and so on. An **Instructor's Manual**, containing full solutions to the Assignments, is available free to lecturers adopting this text (see below).

'Learning by example' is at the heart of '*Higher Engineering Mathematics 5th Edition*'.

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Free web downloads

Extra material available on the Internet

It is recognised that the **level of understanding of algebra** on entry to higher courses is often inadequate. Since algebra provides the basis of so much of higher engineering studies, it is a situation that often needs urgent attention. Lack of space has prevented the inclusion of more basic algebra topics in this textbook;

it is for this reason that some algebra topics – solution of simple, simultaneous and quadratic equations and transposition of formulae have been made available to all via the Internet. Also included is a Remedial Algebra Assignment to test understanding.

To access the Algebra material visit: http://books.elsevier.com/companions/0750681527

Sample of Worked Solutions to Exercises
Within the text are some 1750 further problems arranged within 250 Exercises. A sample of over 1000 worked solutions has been prepared and is available for lecturers only at http://www.textbooks.elsevier.com

Instructor's manual
This provides full worked solutions and mark scheme for all 19 Assignments in this book, together with solutions to the Remedial Algebra Assignment mentioned above. The material is available to lecturers only and is available at http://www.textbooks.elsevier.com

To access the lecturer material on the textbook website please go to http://www.textbooks.elsevier.com and search for the book and click on the 'manual' link. If you do not have an account on textbooks.elsevier.com already, you will need to register and request access to the book's subject area. If you already have an account on textbooks, but do not have access to the right subject area, please follow the 'request access' link at the top of the subject area homepage.

Syllabus guidance

This textbook is written for **undergraduate engineering degree and foundation degree courses**; however, it is also most appropriate for **HNC/D studies** and three syllabuses are covered. The appropriate chapters for these three syllabuses are shown in the table below.

Chapter		**Analytical Methods for Engineers**	**Further Analytical Methods for Engineers**	**Engineering Mathematics**
1.	Algebra	×		
2.	Inequalities			
3.	Partial fractions	×		
4.	Logarithms and exponential functions	×		
5.	Hyperbolic functions	×		
6.	Arithmetic and geometric progressions	×		
7.	The binomial series	×		
8.	Maclaurin's series	×		
9.	Solving equations by iterative methods		×	
10.	Computer numbering systems		×	
11.	Boolean algebra and logic circuits		×	
12.	Introduction to trigonometry	×		
13.	Cartesian and polar co-ordinates	×		
14.	The circle and its properties	×		
15.	Trigonometric waveforms	×		
16.	Trigonometric identities and equations	×		
17.	The relationship between trigonometric and hyperbolic functions	×		
18.	Compound angles	×		
19.	Functions and their curves		×	
20.	Irregular areas, volumes and mean value of waveforms		×	
21.	Vectors, phasors and the combination of waveforms		×	
22.	Scalar and vector products		×	
23.	Complex numbers		×	
24.	De Moivre's theorem		×	
25.	The theory of matrices and determinants		×	
26.	The solution of simultaneous equations by matrices and determinants		×	
27.	Methods of differentiation	×		
28.	Some applications of differentiation	×		
29.	Differentiation of parametric equations			
30.	Differentiation of implicit functions	×		
31.	Logarithmic differentiation	×		
32.	Differentiation of hyperbolic functions	×		
33.	Differentiation of inverse trigonometric and hyperbolic functions	×		
34.	Partial differentiation			×

(*Continued*)

Chapter		Analytical Methods for Engineers	Further Analytical Methods for Engineers	Engineering Mathematics
35.	Total differential, rates of change and small changes			×
36.	Maxima, minima and saddle points for functions of two variables			×
37.	Standard integration	×		
38.	Some applications of integration	×		
39.	Integration using algebraic substitutions	×		
40.	Integration using trigonometric and hyperbolic substitutions	×		
41.	Integration using partial fractions	×		
42.	The $t = \tan \theta/2$ substitution			
43.	Integration by parts	×		
44.	Reduction formulae	×		
45.	Numerical integration		×	
46.	Solution of first order differential equations by separation of variables		×	
47.	Homogeneous first order differential equations			
48.	Linear first order differential equations		×	
49.	Numerical methods for first order differential equations		×	×
50.	Second order differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$		×	
51.	Second order differential equations of the form $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$		×	
52.	Power series methods of solving ordinary differential equations			×
53.	An introduction to partial differential equations			×
54.	Presentation of statistical data	×		
55.	Measures of central tendency and dispersion	×		
56.	Probability	×		
57.	The binomial and Poisson distributions	×		
58.	The normal distribution	×		
59.	Linear correlation	×		
60.	Linear regression	×		
61.	Sampling and estimation theories	×		
62.	Significance testing	×		
63.	Chi-square and distribution-free tests	×		
64.	Introduction to Laplace transforms			×
65.	Properties of Laplace transforms			×
66.	Inverse Laplace transforms			×
67.	Solution of differential equations using Laplace transforms			×
68.	The solution of simultaneous differential equations using Laplace transforms			×
69.	Fourier series for periodic functions of period 2π			×
70.	Fourier series for non-periodic functions over range 2π			×
71.	Even and odd functions and half-range Fourier series			×
72.	Fourier series over any range			×
73.	A numerical method of harmonic analysis			×
74.	The complex or exponential form of a Fourier series			×

1

Algebra

1.1 Introduction

In this chapter, polynomial division and the factor and remainder theorems are explained (in Sections 1.4 to 1.6). However, before this, some essential algebra revision on basic laws and equations is included.

For further Algebra revision, go to website: http://books.elsevier.com/companions/0750681527

1.2 Revision of basic laws

(a) Basic operations and laws of indices

The **laws of indices** are:

(i) $a^m \times a^n = a^{m+n}$ (ii) $\dfrac{a^m}{a^n} = a^{m-n}$

(iii) $(a^m)^n = a^{m \times n}$ (iv) $a^{\frac{m}{n}} = \sqrt[n]{a^m}$

(v) $a^{-n} = \dfrac{1}{a^n}$ (vi) $a^0 = 1$

Problem 1. Evaluate $4a^2bc^3 - 2ac$ when $a = 2$, $b = \frac{1}{2}$ and $c = 1\frac{1}{2}$

$$4a^2bc^3 - 2ac = 4(2)^2\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)^3 - 2(2)\left(\frac{3}{2}\right)$$
$$= \frac{4 \times 2 \times 2 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 2} - \frac{12}{2}$$
$$= 27 - 6 = \mathbf{21}$$

Problem 2. Multiply $3x + 2y$ by $x - y$.

$$\begin{array}{rl} & 3x \ + 2y \\ & x \ - y \\ \hline \text{Multiply by } x \ \rightarrow & 3x^2 + 2xy \\ \text{Multiply by } -y \rightarrow & \quad - 3xy - \ 2y^2 \\ \hline \text{Adding gives:} & \boldsymbol{3x^2 - \ xy - 2y^2} \end{array}$$

Alternatively,

$$(3x + 2y)(x - y) = 3x^2 - 3xy + 2xy - 2y^2$$
$$= \mathbf{3x^2 - xy - 2y^2}$$

Problem 3. Simplify $\dfrac{a^3b^2c^4}{abc^{-2}}$ and evaluate when $a = 3$, $b = \frac{1}{8}$ and $c = 2$.

$$\frac{a^3b^2c^4}{abc^{-2}} = a^{3-1}b^{2-1}c^{4-(-2)} = \boldsymbol{a^2bc^6}$$

When $a = 3$, $b = \frac{1}{8}$ and $c = 2$,

$$a^2bc^6 = (3)^2\left(\tfrac{1}{8}\right)(2)^6 = (9)\left(\tfrac{1}{8}\right)(64) = \mathbf{72}$$

Problem 4. Simplify $\dfrac{x^2y^3 + xy^2}{xy}$

$$\frac{x^2y^3 + xy^2}{xy} = \frac{x^2y^3}{xy} + \frac{xy^2}{xy}$$
$$= x^{2-1}y^{3-1} + x^{1-1}y^{2-1}$$
$$= \boldsymbol{xy^2 + y} \quad \text{or} \quad \boldsymbol{y(xy + 1)}$$

Problem 5. Simplify $\dfrac{(x^2\sqrt{y})(\sqrt{x}\sqrt[3]{y^2})}{(x^5y^3)^{\frac{1}{2}}}$

$$\frac{(x^2\sqrt{y})(\sqrt{x}\sqrt[3]{y^2})}{(x^5y^3)^{\frac{1}{2}}} = \frac{x^2y^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{2}{3}}}{x^{\frac{5}{2}}y^{\frac{3}{2}}}$$
$$= x^{2+\frac{1}{2}-\frac{5}{2}}y^{\frac{1}{2}+\frac{2}{3}-\frac{3}{2}}$$
$$= x^0y^{-\frac{1}{3}}$$
$$= \boldsymbol{y^{-\frac{1}{3}}} \quad \text{or} \quad \boldsymbol{\frac{1}{y^{\frac{1}{3}}}} \quad \text{or} \quad \boldsymbol{\frac{1}{\sqrt[3]{y}}}$$

Now try the following exercise.

Exercise 1 Revision of basic operations and laws of indices

1. Evaluate $2ab + 3bc - abc$ when $a = 2$, $b = -2$ and $c = 4$. $[-16]$
2. Find the value of $5pq^2r^3$ when $p = \frac{2}{5}$, $q = -2$ and $r = -1$. $[-8]$
3. From $4x - 3y + 2z$ subtract $x + 2y - 3z$. $[3x - 5y + 5z]$
4. Multiply $2a - 5b + c$ by $3a + b$. $[6a^2 - 13ab + 3ac - 5b^2 + bc]$
5. Simplify $(x^2y^3z)(x^3yz^2)$ and evaluate when $x = \frac{1}{2}$, $y = 2$ and $z = 3$. $[x^5y^4z^3, 13\frac{1}{2}]$
6. Evaluate $(a^{\frac{3}{2}}bc^{-3})(a^{\frac{1}{2}}b^{-\frac{1}{2}}c)$ when $a = 3$, $b = 4$ and $c = 2$. $[\pm 4\frac{1}{2}]$
7. Simplify $\dfrac{a^2b + a^3b}{a^2b^2}$ $\left[\dfrac{1+a}{b}\right]$
8. Simplify $\dfrac{(a^3b^{\frac{1}{2}}c^{-\frac{1}{2}})(ab)^{\frac{1}{3}}}{(\sqrt{a^3}\sqrt{b}\,c)}$ $\left[a^{\frac{11}{6}}b^{\frac{1}{3}}c^{-\frac{3}{2}} \quad \text{or} \quad \dfrac{\sqrt[6]{a}^{11}\sqrt[3]{b}}{\sqrt{c^3}}\right]$

(b) Brackets, factorization and precedence

Problem 6. Simplify
$$a^2 - (2a - ab) - a(3b + a).$$

$$\begin{aligned} &a^2 - (2a - ab) - a(3b + a) \\ &= a^2 - 2a + ab - 3ab - a^2 \\ &= \mathbf{-2a - 2ab} \quad \text{or} \quad \mathbf{-2a(1 + b)} \end{aligned}$$

Problem 7. Remove the brackets and simplify the expression:
$$2a - [3\{2(4a - b) - 5(a + 2b)\} + 4a].$$

Removing the innermost brackets gives:
$$2a - [3\{8a - 2b - 5a - 10b\} + 4a]$$

Collecting together similar terms gives:
$$2a - [3\{3a - 12b\} + 4a]$$

Removing the 'curly' brackets gives:
$$2a - [9a - 36b + 4a]$$

Collecting together similar terms gives:
$$2a - [13a - 36b]$$

Removing the square brackets gives:
$$2a - 13a + 36b = \mathbf{-11a + 36b} \quad \text{or} \quad \mathbf{36b - 11a}$$

Problem 8. Factorize (a) $xy - 3xz$ (b) $4a^2 + 16ab^3$ (c) $3a^2b - 6ab^2 + 15ab$.

(a) $xy - 3xz = \mathbf{x(y - 3z)}$

(b) $4a^2 + 16ab^3 = \mathbf{4a(a + 4b^3)}$

(c) $3a^2b - 6ab^2 + 15ab = \mathbf{3ab(a - 2b + 5)}$

Problem 9. Simplify $3c + 2c \times 4c + c \div 5c - 8c$.

The order of precedence is division, multiplication, addition and subtraction (sometimes remembered by BODMAS). Hence

$$\begin{aligned} &3c + 2c \times 4c + c \div 5c - 8c \\ &= 3c + 2c \times 4c + \left(\frac{c}{5c}\right) - 8c \\ &= 3c + 8c^2 + \frac{1}{5} - 8c \\ &= \mathbf{8c^2 - 5c + \frac{1}{5}} \quad \text{or} \quad \mathbf{c(8c - 5) + \frac{1}{5}} \end{aligned}$$

Problem 10. Simplify $(2a - 3) \div 4a + 5 \times 6 - 3a$.

$$\begin{aligned} &(2a - 3) \div 4a + 5 \times 6 - 3a \\ &= \frac{2a - 3}{4a} + 5 \times 6 - 3a \\ &= \frac{2a - 3}{4a} + 30 - 3a \\ &= \frac{2a}{4a} - \frac{3}{4a} + 30 - 3a \\ &= \frac{1}{2} - \frac{3}{4a} + 30 - 3a = \mathbf{30\frac{1}{2} - \frac{3}{4a} - 3a} \end{aligned}$$

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Now try the following exercise.

Exercise 2 Further problems on brackets, factorization and precedence

1. Simplify $2(p+3q-r)-4(r-q+2p)+p$. $[-5p+10q-6r]$
2. Expand and simplify $(x+y)(x-2y)$. $[x^2-xy-2y^2]$
3. Remove the brackets and simplify: $24p-[2\{3(5p-q)-2(p+2q)\}+3q]$. $[11q-2p]$
4. Factorize $21a^2b^2-28ab$ $[7ab(3ab-4)]$.
5. Factorize $2xy^2+6x^2y+8x^3y$. $[2xy(y+3x+4x^2)]$
6. Simplify $2y+4\div 6y+3\times 4-5y$. $\left[\frac{2}{3y}-3y+12\right]$
7. Simplify $3\div y+2\div y-1$. $\left[\frac{5}{y}-1\right]$
8. Simplify $a^2-3ab\times 2a\div 6b+ab$. $[ab]$

1.3 Revision of equations

(a) Simple equations

Problem 11. Solve $4-3x=2x-11$.

Since $4-3x=2x-11$ then $4+11=2x+3x$

i.e. $15=5x$ from which, $\boldsymbol{x=\frac{15}{5}=3}$

Problem 12. Solve $4(2a-3)-2(a-4)=3(a-3)-1$.

Removing the brackets gives:

$$8a-12-2a+8=3a-9-1$$

Rearranging gives:

$$8a-2a-3a=-9-1+12-8$$

i.e. $3a=-6$

and $\boldsymbol{a=\frac{-6}{3}=-2}$

Problem 13. Solve $\frac{3}{x-2}=\frac{4}{3x+4}$.

By 'cross-multiplying': $3(3x+4)=4(x-2)$

Removing brackets gives: $9x+12=4x-8$

Rearranging gives: $9x-4x=-8-12$

i.e. $5x=-20$

and $\boldsymbol{x=\frac{-20}{5}=-4}$

Problem 14. Solve $\left(\frac{\sqrt{t}+3}{\sqrt{t}}\right)=2$.

$$\sqrt{t}\left(\frac{\sqrt{t}+3}{\sqrt{t}}\right)=2\sqrt{t}$$

i.e. $\sqrt{t}+3=2\sqrt{t}$

and $3=2\sqrt{t}-\sqrt{t}$

i.e. $3=\sqrt{t}$

and $\boldsymbol{9=t}$

(b) Transposition of formulae

Problem 15. Transpose the formula $v=u+\frac{ft}{m}$ to make f the subject.

$u+\frac{ft}{m}=v$ from which, $\frac{ft}{m}=v-u$

and $m\left(\frac{ft}{m}\right)=m(v-u)$

i.e. $ft=m(v-u)$

and $\boldsymbol{f=\frac{m}{t}(v-u)}$

Problem 16. The impedance of an a.c. circuit is given by $Z=\sqrt{R^2+X^2}$. Make the reactance X the subject.

$\sqrt{R^2+X^2}=Z$ and squaring both sides gives $R^2+X^2=Z^2$, from which,

$X^2=Z^2-R^2$ and **reactance** $\boldsymbol{X=\sqrt{Z^2-R^2}}$

Problem 17. Given that $\dfrac{D}{\mathrm{d}}=\sqrt{\left(\dfrac{f+p}{f-p}\right)}$, express p in terms of D, d and f.

Rearranging gives: $\sqrt{\left(\dfrac{f+p}{f-p}\right)}=\dfrac{D}{\mathrm{d}}$

Squaring both sides gives: $\dfrac{f+p}{f-p}=\dfrac{D^2}{\mathrm{d}^2}$

'Cross-multiplying' gives:

$$\mathrm{d}^2(f+p)=D^2(f-p)$$

Removing brackets gives:

$$\mathrm{d}^2f+\mathrm{d}^2p=D^2f-D^2p$$

Rearranging gives: $\mathrm{d}^2p+D^2p=D^2f-\mathrm{d}^2f$

Factorizing gives: $p(\mathrm{d}^2+D^2)=f(D^2-\mathrm{d}^2)$

and $$\boldsymbol{p=\dfrac{f(D^2-\mathrm{d}^2)}{(\mathrm{d}^2+D^2)}}$$

Now try the following exercise.

Exercise 3 Further problems on simple equations and transposition of formulae

In problems 1 to 4 solve the equations

1. $3x-2-5x=2x-4$ $\left[\frac{1}{2}\right]$
2. $8+4(x-1)-5(x-3)=2(5-2x)$ $[-3]$
3. $\dfrac{1}{3a-2}+\dfrac{1}{5a+3}=0$ $\left[-\frac{1}{8}\right]$
4. $\dfrac{3\sqrt{t}}{1-\sqrt{t}}=-6$ $[4]$
5. Transpose $y=\dfrac{3(F-f)}{L}$ for f. $\left[f=\dfrac{3F-yL}{3} \quad \text{or} \quad f=F-\dfrac{yL}{3}\right]$
6. Make l the subject of $t=2\pi\sqrt{\dfrac{l}{g}}$ $\left[l=\dfrac{t^2g}{4\pi^2}\right]$
7. Transpose $m=\dfrac{\mu L}{L+rCR}$ for L. $\left[L=\dfrac{mrCR}{\mu-m}\right]$
8. Make r the subject of the formula $\dfrac{x}{y}=\dfrac{1+r^2}{1-r^2}$ $\left[r=\sqrt{\left(\dfrac{x-y}{x+y}\right)}\right]$

(c) Simultaneous equations

Problem 18. Solve the simultaneous equations:

$$7x-2y=26 \quad (1)$$

$$6x+5y=29 \quad (2)$$

$5\times$ equation (1) gives:

$$35x-10y=130 \quad (3)$$

$2\times$ equation (2) gives:

$$12x+10y=58 \quad (4)$$

equation (3) + equation (4) gives:

$$47x+0=188$$

from which, $\boldsymbol{x=\dfrac{188}{47}=4}$

Substituting $x=4$ in equation (1) gives:

$$28-2y=26$$

from which, $28-26=2y$ and $\boldsymbol{y=1}$

Problem 19. Solve

$$\frac{x}{8}+\frac{5}{2}=y \quad (1)$$

$$11+\frac{y}{3}=3x \quad (2)$$

$8\times$ equation (1) gives: $x+20=8y$ (3)

$3\times$ equation (2) gives: $33+y=9x$ (4)

i.e. $x-8y=-20$ (5)

A

and $9x - y = 33$ (6)

$8 \times$ equation (6) gives: $72x - 8y = 264$ (7)

Equation (7) − equation (5) gives:

$$71x = 284$$

from which, $\boldsymbol{x = \dfrac{284}{71} = 4}$

Substituting $x = 4$ in equation (5) gives:

$$4 - 8y = -20$$

from which, $4 + 20 = 8y$ and $\boldsymbol{y = 3}$

(d) Quadratic equations

Problem 20. Solve the following equations by factorization:

(a) $3x^2 - 11x - 4 = 0$

(b) $4x^2 + 8x + 3 = 0$

(a) The factors of $3x^2$ are $3x$ and x and these are placed in brackets thus:
$(3x \quad)(x \quad)$
The factors of -4 are $+1$ and -4 or -1 and $+4$, or -2 and $+2$. Remembering that the product of the two inner terms added to the product of the two outer terms must equal $-11x$, the only combination to give this is $+1$ and -4, i.e.,

$$3x^2 - 11x - 4 = (3x + 1)(x - 4)$$

Thus $(3x + 1)(x - 4) = 0$ hence

either $(3x + 1) = 0$ i.e. $\boldsymbol{x = -\frac{1}{3}}$

or $(x - 4) = 0$ i.e. $\boldsymbol{x = 4}$

(b) $4x^2 + 8x + 3 = (2x + 3)(2x + 1)$

Thus $(2x + 3)(2x + 1) = 0$ hence

either $(2x + 3) = 0$ i.e. $\boldsymbol{x = -\frac{3}{2}}$

or $(2x + 1) = 0$ i.e. $\boldsymbol{x = -\frac{1}{2}}$

Problem 21. The roots of a quadratic equation are $\frac{1}{3}$ and -2. Determine the equation in x.

If $\frac{1}{3}$ and -2 are the roots of a quadratic equation then,

$$(x - \tfrac{1}{3})(x + 2) = 0$$

i.e. $x^2 + 2x - \frac{1}{3}x - \frac{2}{3} = 0$

i.e. $x^2 + \frac{5}{3}x - \frac{2}{3} = 0$

or $\boldsymbol{3x^2 + 5x - 2 = 0}$

Problem 22. Solve $4x^2 + 7x + 2 = 0$ giving the answer correct to 2 decimal places.

From the quadratic formula if $ax^2 + bx + c = 0$ then,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hence if $4x^2 + 7x + 2 = 0$

then $x = \dfrac{-7 \pm \sqrt{7^2 - 4(4)(2)}}{2(4)}$

$$= \frac{-7 \pm \sqrt{17}}{8}$$

$$= \frac{-7 \pm 4.123}{8}$$

$$= \frac{-7 + 4.123}{8} \text{ or } \frac{-7 - 4.123}{8}$$

i.e. $\boldsymbol{x = -0.36}$ or $\boldsymbol{-1.39}$

Now try the following exercise.

Exercise 4 Further problems on simultaneous and quadratic equations

In problems 1 to 3, solve the simultaneous equations

1. $8x - 3y = 51$
   $3x + 4y = 14$ [$x = 6$, $y = -1$]

2. $5a = 1 - 3b$
   $2b + a + 4 = 0$ [$a = 2$, $b = -3$]

3. $\dfrac{x}{5} + \dfrac{2y}{3} = \dfrac{49}{15}$
   $\dfrac{3x}{7} - \dfrac{y}{2} + \dfrac{5}{7} = 0$ [$x = 3$, $y = 4$]

4. Solve the following quadratic equations by factorization:
   (a) $x^2 + 4x - 32 = 0$
   (b) $8x^2 + 2x - 15 = 0$
   [(a) 4, -8 (b) $\frac{5}{4}$, $-\frac{3}{2}$]