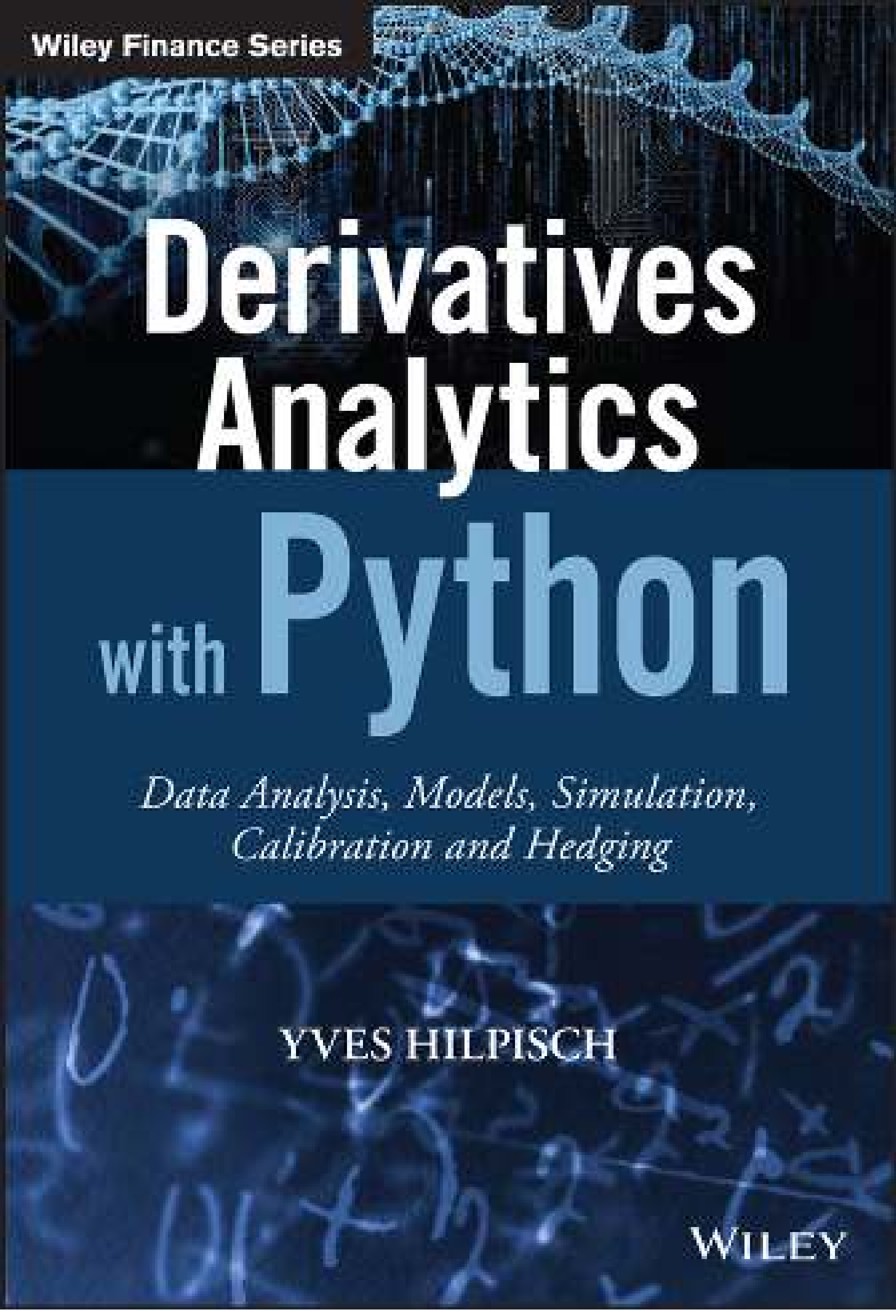
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Data Analysis, Models, Simulation, Calibration and Hedging
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Preface

This book is an outgrowth of diverse activities of myself and colleagues of mine in the fields of financial engineering, computational finance and Python programming at our company The Python Quants GmbH on the one hand and of teaching mathematical finance at Saarland University on the other hand.

The book is targeted at practitioners, researchers and students interested in the market-based valuation of options from a practical perspective, i.e. the single numerical and technical implementation steps that make up such an effort. It is also for those who want to learn how Python can be used for derivatives analytics and financial engineering. However, apart from being primarily practical and implementation-oriented, the book also provides the necessary theoretical foundations and numerical tools.

My hope is that the book will contribute to the increasing acceptance of Python in the financial community, and in particular in the analytics space. If you are interested in getting the Python scripts and IPython Notebooks accompanying the book, you should visit http://wiley.quant-platform.com where you can register for the Quant Platform which allows browser-based, interactive and collaborative financial analytics. Further resources are found on the website http://derivatives-analytics-with-python.com. You should also check out the open source library DX Analytics under http://dx-analytics.com which implements the concepts and methods presented in the book in standardized, reusable fashion.

I thank my family—and in particular my wife Sandra—for their support and understanding that such a project requires many hours of solitude. I also want to thank my colleague Michael Schwed for his continuous help and support. In addition, I thank Alain Ledon and Riaz Ahmad for their comments and feedback. Discussions with participants of seminars and my lectures at Saarland University also helped the project significantly. Parts of this book have benefited from talks I have given at diverse Python and finance conferences over the years.

I dedicate this book to my lovely son Henry Nikolaus whose direct approach to living and clear view of the world I admire.

YVES HILPISCH
Saarland, February 2015

CHAPTER 1

A Quick Tour

1.1 MARKET-BASED VALUATION

This book is about the market-based valuation of (stock) index options. In the domain of derivatives analytics this is an important task which every major investment bank and buy-side decision maker in the financial market is concerned with on a daily basis. While theoretical valuation approaches develop a model, parametrize it and then derive values for options, the market-based approach works the other way round. Prices from liquidly traded options are taken as given (i.e. they are inputs instead of outputs) and one tries to parametrize a market model in a way that replicates the observed option prices as well as possible. This activity is generally referred to as *model calibration*. Being equipped with a calibrated model, one then proceeds with the task at hand, be it valuation, trading, investing, hedging or risk management. A bit more specifically, one might be interested in pricing and hedging an exotic derivative instrument with such a model—hoping that the results are in line with the overall market (i.e. arbitrage-free and even "fair") due to the previous calibration to more simple, vanilla instruments.

To accomplish a market-based valuation, four areas have to be covered:

1. **market**: knowledge about market realities is a conditio sine qua non for any sincere attempt to develop market-consistent models and to accomplish market-based valuation
2. **theory**: every valuation must be grounded on a sound market model, ensuring, for example, the absence of arbitrage opportunities and providing means to derive option values from observed quantities
3. **numerics**: one cannot hope to work with analytical results only; numerical techniques, like Monte Carlo simulation, are generally required in different steps of a market-based valuation process
4. **technology**: to implement numerical techniques efficiently, one is dependent on appropriate technology (hard- and software)

This book covers all of these areas in an integrated manner. It uses equity index options as the prime example for derivative instruments throughout. This, among others, allows to abstract from dividend related issues.

1.2 STRUCTURE OF THE BOOK

The book is divided into three parts. The first part is concerned with market-based valuation as a process and empirical findings about market realities. The second part covers a number of topics for the theoretical valuation of options and derivatives. It also develops tools much needed during a market-based valuation. The third part finally covers the major aspects related to a market-based valuation and also hedging strategies in such a context.

Part I "The Market" comprises two chapters:

- **Chapter 2**: this chapter contains a discussion of topics related to market-based valuation, like risks affecting the value of equity index options
- **Chapter 3**: this chapter documents empirical and anecdotal facts about stocks, stock indices and in particular volatility (e.g. stochasticity, clustering, smiles) as well as about interest rates

Part II "Theoretical Valuation" comprises four chapters:

- **Chapter 4**: this chapter covers arbitrage pricing theory and risk-neutral valuation in discrete time (in some detail) and continuous time (on a higher level) according to the Harrison-Kreps-Pliska paradigm (cf. Harrison and Kreps (1979) and Harrison and Pliska (1981))
- **Chapter 5**: the topic of this chapter is the complete market models of Black-Scholes-Merton (BSM, cf. Black and Scholes (1973), Merton (1973)) and Cox-Ross-Rubinstein (CRR, cf. Cox et al. (1979)) that are generally considered benchmarks for option valuation
- **Chapter 6**: Fourier-based approaches allow us to derive semi-analytical valuation formulas for European options in market models more complex and realistic than the BSM/CRR models; this chapter introduces the two popular methods of Carr-Madan (cf. Carr and Madan (1999)) and Lewis (cf. Lewis (2001))
- **Chapter 7**: the valuation of American options is more involved than with European options; this chapter analyzes the respective problem and introduces algorithms for American option valution via binomial trees and Monte Carlo simulation; at the center stands the Least-Squares Monte Carlo algorithm of Longstaff-Schwartz (cf. Longstaff and Schwartz (2001))

Finally, **Part III "Market-Based Valuation"** has seven chapters:

- **Chapter 8**: before going into details, this chapter illustrates the whole process of a market-based valuation effort in the simple, but nevertheless still useful, setting of Merton's jump-diffusion model (cf. Merton (1976))
- **Chapter 9**: this chapter introduces the general market model used henceforth, which is from Bakshi-Cao-Chen (cf. Bakshi et al. (1997)) and which accounts for stochastic volatility, jumps and stochastic short rates
- **Chapter 10**: Monte Carlo simulation is generally the method of choice for the valuation of exotic/complex index options and derivatives; this chapter therefore discusses in some detail the discretization and simulation of the stochastic volatility model by Heston

(cf. Heston (1993)) with constant as well as stochastic short rates according to Cox-Ingersoll-Ross (cf. Cox et al. (1985))
- **Chapter 11**: model calibration stays at the center of market-based valuation; the chapter considers several general aspects associated with this topic and then proceeds with the numerical calibration of the general market model to real market data
- **Chapter 12**: this chapter combines the results from the previous two to value European and American index options via Monte Carlo simulation in the calibrated general market model
- **Chapter 13**: this chapter analyzes dynamic delta hedging strategies for American options by Monte Carlo simulation in different settings, from a simple one to the calibrated market model
- **Chapter 14**: this brief chapter provides a concise summary of central aspects related to the market-based valuation of index options

In addition, the book has an **Appendix** with one chapter:

- **Appendix A**: the appendix introduces some of the most important Python concepts and libraries in a nutshell; the selection of topics is clearly influenced by the requirements of the rest of the book; those not familiar with Python or looking for details should consult the more comprehensive treatment of all relevant topics by the same author (cf. Hilpisch (2014))

1.3 WHY PYTHON?

Although Python has established itself in the financial industry as a powerful programming language with an elaborate ecosystem of tools and libraries, it is still not often used for financial, derivatives or risk analytics purposes. Languages like C++, C, C#, VBA or Java and toolboxes like Matlab or domain-specific languages like R often dominate this area. However, we see a number of good reasons to choose Python even for computationally demanding analytics tasks; the following are the most important ones we want to mention, in no particular order, (see also chapter 1 in Hilpisch (2014)):

- **open source**: Python and the majority of available libraries are completely open source; this allows an entry to this technology at no cost, something particularly important for students, academics or other individuals
- **syntax**: Python programming is easy to learn, the code is quite compact and in general highly readable; at universities it is increasingly used as an introduction to programming in general; when it comes to numerical or financial algorithm implementation, the syntax is pretty close to the mathematics in general (e.g. due to code vectorization approaches)
- **multi-paradigm**: Python is as good for procedural programming (which suffices for the purposes of this book) as well as at object-oriented programming (which is necessary in more complex/professional contexts); it also has some functional programming features to offer
- **interpreted**: Python is an interpreted language which makes rapid prototyping and development in general a bit more convenient, especially for beginners; tools like IPython

Notebook and libraries like pandas for time series analysis allow for efficient and productive interactive analytics workflows
- **libraries**: nowadays, there is a wealth of powerful libraries available and the supply grows steadily; there is hardly a problem that cannot be easily tackled with an existing library, be it a numerical problem, a graphical one or a data-related problem
- **speed**: a common prejudice with regard to interpreted languages—compared to compiled ones like C++ or C—is the slow speed of code execution; however, financial applications are more or less all about matrix and array manipulations and operations which can be done at the speed of C code with the essential Python library NumPy for array-based computing; other performance libraries, like Numba for dynamic code compiling, can also be used to improve performance
- **market**: in the London area (mainly financial services) the number of Python developer contract offerings was 485 in the third quarter of 2012; the comparable figure in the same period 2013 was already 864;[1] large financial institutions like Bank of America, Merrill Lynch and J.P. Morgan have millions of lines of Python code in production, mainly in risk management; Python is also really popular in the hedge fund industry

All in all, Python seems to be a good choice for our purposes. The cover story “Python Takes a Bite” in the March 2010 issue of *Wilmott* magazine (cf. Lee (2010)) also illustrates that Python is gaining ground in the financial world. A modern introduction into Python for finance is given by Hilpisch (2014).

One of the easiest ways to get started with Python is to register on the *Quant Platform* which allows for browser-based, interactive and collaborative financial analytics and development (cf. http://quant-platform.com). This platform offers all you need to do efficient and productive financial analytics as well as financial application building with Python. It also provides, for instance, integration with R, the free software environment for statistical computing and graphics.

1.4 FURTHER READING

The book covers a great variety of aspects which comes at the cost of depth of exposition and analysis in some places. Our aim is to emphasize the red line and to guide the reader easily through the different topics. However, this inevitably leads to uncovered aspects, omitted proofs and unanswered questions. Fortunately, a number of good sources in book form are available which may be consulted on the different topics:

- **market**: cf. Bittmann (2009) to learn about options fundamentals, the main microstructure elements of their markets and the specific lingo; Gatheral (2006) is a concise reference about option and volatility modeling in practice; Rebonato (2004) is a book that comprehensively covers option markets, their empirical specialities and the models used in theory and practice

[1] Source: www.itjobswatch.co.uk/contracts/london/python.do on 07. October 2014.